Subject	class	Paper	Study materials on	Resource Person
Mathematics	D-I (H)	I	Transformation of biquadratic eqn into the form $z^4+6Hz^2+4Gz+(a_0^2I-3H^2)$	Dr. S. Ahmad

Q. Reduce the biquadratic equation $a_0x^4+4a_1x^3+6a_2x^2+4a_3x+a_4=0$ in the form $z^4+6Hz^2+4Gz+(a_0^2I-3H^2)=0$

Where G, H, I have their usual meaning.

Soln. The given equation is

$$f(x)\equiv a_0x^4+4a_1x^3+6a_2x^2+4a_3x+a_4=0 \quad \text{——— (1)}$$

In order to remove the 2nd term, let us diminish the roots of (1) by h and resulting equation is obtained by putting $y=x-h$ or $x=y+h$ in (1), therefore

$$a_0(y+h)^4+4a_1(y+h)^3+6a_2(y+h)^2+4a_3(y+h)+a_4=0$$

$$\Rightarrow a_0(y^4+4y^3h+6y^2h^2+4yh^3+h^4)+4a_1(y^3+3y^2h+3yh^2+h^3)$$
$$+6a_2(y^2+2yh+h^2)+4a_3(y+h)+a_4=0$$

$$\Rightarrow a_0y^4+4(a_0h+a_1)y^3+6(a_0h^2+2a_1h+a_2)y^2+4(a_0h^3+3a_1h^2+2a_2h+a_3)y$$
$$+(a_0h^4+4a_1h^3+6a_2h^2+4a_3h+a_4)=0$$

$$\Rightarrow A_0y^4+4A_1y^3+6A_2y^2+4A_3y+A_4=0$$

Where $A_0=a_0$, $A_1=a_0h+a_1$, $A_2=a_0h^2+2a_1h+a_2$

$A_3=a_0h^3+3a_1h^2+2a_2h+a_3$

$A_4=a_0h^4+4a_1h^3+6a_2h^2+4a_3h+a_4$

If the 2nd term is to be removed, then $A_1=0\Rightarrow a_0h+a_1=0$

$\therefore h=\frac{-a_1}{a_0}$.

We can easily calculate by putting the value of $h=\frac{-a_1}{a_0}$ in A_2 & A_3 & thus $A_2=\frac{H}{a_0}$, $A_3=\frac{G}{a_0^2}$

$$A_4=a_0h^4+4a_1h^3+6a_2h^2+4a_3h+a_4$$
$$=a_0\left(\frac{-a_1}{a_0}\right)^4+4a_1\left(\frac{-a_1}{a_0}\right)^3+6a_2\left(\frac{-a_1}{a_0}\right)^2+4a_3\left(\frac{-a_1}{a_0}\right)+a_4$$

$$= \frac{a_0^3 a_4 - 4a_0^2 a_1 a_3 + 6a_0 a_1^2 a_2 - 3a_1^4}{a_0^3}$$

$$= \frac{a_0^2(a_0 a_4 - 4a_1 a_3 + 3a_2^2) - 3(a_0 a_2 - a_1^2)^2}{a_0^3}$$

$$= \frac{a_0^2 I - 3H^2}{a_0^3}$$

and $A_0 = a_0$

Hence the transformed equation is

$$a_0 y^4 + \frac{6H}{a_0} y^2 + \frac{4G}{a_0^2} y + \frac{a_0^2 I - 3H^2}{a_0^3} = 0$$

$$\Rightarrow y^4 + \frac{6H}{a_0^2} y^2 + \frac{4G}{a_0^3} y + \frac{a_0^2 I - 3H^2}{a_0^4} = 0 \quad \text{--- (2)}$$

Now, we want that the coefficients in (2) should be integral and we should multiply its roots by a_0 and then transformed eqn is

$$z^4 + 0.a_0 z^3 + a_0^2 \frac{6H}{a_0^2} z^2 + a_0^3 . \frac{4G}{a_0^3} . z + a_0^4 . \frac{a_0^2 I - 3H^2}{a_0^4} = 0$$

$$\Rightarrow \boxed{z^4 + 6Hz^2 + 4Gz + (a_0^2 I - 3H^2) = 0}$$

It is required form.